PG MA S1- 03 (DSC)

# TWO YEAR MASTER'S DEGREE COURSE

**SEMESTER - I**

## Real Analysis

# MATHEMATICS

**Discipline Specific Core Course**
**MASTER OF SCIENCE IN MATHEMATICS**

**Hiranya Chandra Bhuyan School of Science and Technology**

**KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY**

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# CONTENTS


| | | PAGES |
|---|---|---|
| **UNIT 1 :** | **Real Number System-I**<br>Field axioms, some properties of real numbers, Absolute value or modulus of real number, Bounded and unbounded subsets of real numbers, least upper bound or supremum, Greatest lower bound or infimum, Some properties of supremum and infimum, Completeness axiom, The set of real number as a complete ordered field, Archimedean property of real numbers, The denseness property of the real number system. | **7-47** |
| **UNIT 2 :** | **Real Number System-II**<br>Neighbourhood of a point, Limit points of a set, Bolzano Weierstrass theorem, Interior of a set, Open sets, Closed sets, Closure of a set, Countability of sets, Dedekind's property of real numbers. | **48-80** |
| **UNIT 3 :** | **Sequence**<br>Sequence, Subsequence, Bounded sequences, Convergent sequences, Divergent Sequences, Algebra of convergent sequences, Monotonic sequences, Limit point of a sequence, Cauchy's sequences, Cauchy's general principle of convergence, Limit superior and limit inferior of a sequence, Nested interval theorem or Cantor's intersection theorem. | **81-107** |
| **UNIT 4 :** | **Infinite Series-I**<br>Convergent and divergent series, Cauchy's general principle of convergence for series, The auxiliary Series $\sum\left(\frac{1}{n^p}\right)$. | **108-129** |
| **UNIT 5 :** | **Infinite Series-II**<br>Comparison test, Cauchy's root test, D'Alembert's ratio test, Cauchy's condensation test, Raabe's test, Logarithmic test, De Morgan's test and Bertrand's test, Gauss's test, Cauchy Maclaurin's integral test, Alternating series, Alternating series test (Leibnitz's test), Absolute convergence and conditional convergence. | **130-160** |
| **UNIT 6 :** | **Power Series**<br>Power series, Radius of convergence, Uniqueness of power series, Properties of power series. | **161-177** |

**UNIT 7 : Limits of Functions** **178-200**

Limit, Algebra of limits of functions, Right hand and left-hand limits, Infinite limit.

**UNIT 8 : Continuous Functions** **201-217**

Cauchy's definition of continuity, Types of discontinuity, Algebra of continuous functions, Properties of continuous functions, Uniform continuity.

**UNIT 9: Differentiable Functions** **218-240**

Derivative of a point, Derivative of a function, A necessary condition for the existence of finite derivative, Algebra of derivatives, Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem, Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, Taylor's series, Maclaurin's series.

**UNIT 10: Functions of Several Variables** **241-259**

Structure of $R^n$,Limit,continuity of functions of to ,Directional derivatives and Partial derivatives.

**UNIT 11: The Riemann Integral** **260-279**

Partitions and Riemann sums, Upper and lower Riemann integrals, Riemann integrability, Riemann's necessary band sufficient conditions for R-integrability, Some classes of integrable functions, Fundamental theorem of integral calculus.

**Unit 12: The Riemann-Stieltjes integral** **280-292**

A generalization of the Riemann integral, Partitions, Lower and upper Riemann-Stieltjes integrals, The Riemann-Stieltjes integral, The RS-integrals as a limit of sums, Some classes of RS-integrable functions, A relation between R-integral and RS-integral.

**Unit 13: Uniform convergence of sequences and series** **293-312**

Uniform convergence, Cauchy's general principle of uniform convergence, Tests for uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation.

**Unit 14: Convergence of Improper integrals** **313-337**

Convergence of improper integrals, Tests for convergence of improper integrals of the first kind, Absolute convergence, Tests for convergence of improper integrals of the second kind.

**References (For All Units of the Course)** **338**

# COURSE INTRODUCTION

"Real analysis" is the third Discipline Specific core course of MSc (Mathematics) programme. Real analysis is a foundational branch of mathematics that deals with the rigorous study of real numbers, sequences, series, limits, continuity, differentiation, integration, and more. It provides the theoretical underpinnings for calculus and forms the basis for many other areas of mathematics.This course is designed to introduce you to the key concepts and techniques of real analysis, building your understanding from the basics to more advanced topics. Each unit will focus on specific aspects of real analysis, providing theoretical insights, proofs, and examples to illustrate the concepts introduced.

**Unit 1: Real Number System-I**

We begin by examining the foundational properties of the real number system, including field axioms, absolute value, supremum, infimum, completeness axiom, and various properties of real numbers.

**Unit 2: Real Number System-II**

This unit explores concepts such as neighborhoods of points, limit points, open and closed sets, countability of sets, and Dedekind's property of real numbers.

**Unit 3: Sequence**

Sequences are a fundamental concept in real analysis. Here, we study boundedness, convergence, divergence, Cauchy sequences, limit superior and limit inferior, and more.

**Unit 4: Infinite Series-I**

This unit focuses on the convergence and divergence of infinite series, including Cauchy's general principle of convergence and auxiliary series.

**Unit 5: Infinite Series-II**

Further exploring convergence tests for series, this unit covers comparison tests, root and ratio tests, condensation test, logarithmic test, and alternating series.

**Unit 6: Power Series**

Power series are a powerful tool in mathematical analysis. We discuss their properties, radius of convergence, and applications.

**Unit 7: Limits of Functions**

This unit focuses on limits of functions, including right-hand and left-hand limits, infinite limits, and algebraic properties of limits.

**Unit 8: Continuous Functions**

Continuity is a central concept in real analysis. We explore different types of continuity, discontinuities, and properties of continuous functions.

**Unit 9: Differentiable Functions**

We delve into the theory of differentiation, including derivatives, mean value theorems, Taylor's theorem, and applications.

**Unit 10: Functions of Several Variables**

Extending our understanding to functions of multiple variables, we discuss limits, continuity, and partial derivatives.

**Unit 11: The Riemann Integral**

The Riemann integral is a key concept in calculus. Here, we explore partitions, Riemann sums, integrability, and the fundamental theorem of integral calculus.

**Unit 12: The Riemann-Stieltjes integral**

We generalize the Riemann integral to include integrals with respect to a differentiable function, studying partitions, integrability conditions, and relationships with the Riemann integral.

**Unit 13: Uniform convergence of sequences and series**

Uniform convergence of sequences and series is explored, along with its implications for continuity, integration, and differentiation.

**Unit 14: Convergence of Improper integrals**

Finally, we study convergence criteria for improper integrals and absolute convergence.

Each unit includes some along-side boxes to help you know some of the difficult, unseen terms. Some "EXERCISES" have been included to help you apply your own thoughts. You may find some boxes marked with: "LET US KNOW". These boxes will provide you with some additional interesting and relevant information. Again, you will get "CHECK YOUR PROGRESS" questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with "ANSWERS TO CHECK YOUR PROGRESS" given at the end of each unit.